CHALMERS
EXAMINATION / TENTAMEN

Course code/kurskod	Course name/kursnamn			
DAT060-	Logic in Computer Science			
Anonymous code Anonym kod		Examination date Tentamensdatum	Number of pages Antal blad	Grade Betyg
DAT060-25		2016-10-25	8	4

Solved task Behandlade uppgifter No/nr		Points per task Poäng på uppgiften	Observe: Areas with bold contour are to completed by the teacher. Anmärkning: Rutor inom bred kontur ifylles av lärare.
1	X	6.5	
2	X	4	
3	X	3	
4	X	9	
5	X	4	
6	X	2.5	
7	X	3	
8	X	4	
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
Total examination points Summa poäng på tentamen		36	+5.5

Family name+First name (Blockletters)
Efternamn+Förnamn+Initialer(textas)
THORSELL ERIK E.T

Signature
Namnteckning

Year of Admission
Antagningsår

Programme acronym
Program

Identification no
nummer

Date of Birth Year Month Day
Personnummer år mån dag

a) $p \to q, q \to r, p \to s \vdash p \to (r \wedge s)$

1	$p \to q$	premise
2	$q \to r$	premise
3	$p \to s$	premise
4	p	assumption
5	q	$\to e$ 4, 1
6	r	$\to e$ 5, 2
7	s	$\to e$ 4, 3
8	$r \wedge s$	$\wedge i$ 6, 7
9	$p \to (r \wedge s)$	$\to i$ 4-8

√

b) $\neg(p \vee q) \vdash \neg p \wedge \neg q$

1	$\neg(p \vee q)$	premise
2	$p \vee \neg p$	LEM
3	p	?
4	$p \vee q$	
5	$\bot$	
6	$\neg p \wedge \neg q$	
7		

-2.5

c) $p \vee q \vdash \neg q \to p$

1	$p \vee q$	premise
2	$\neg q$	assumption
3	p	assumption
4	q	assumption
5	$\bot$	$\neg e$ 4, 2
6	p	$\bot e$ 5
7	p	$\vee e$ 1, 3-3, 4-6
8	$\neg q \to p$	$\to i$ 2-7

√

CHALMERS

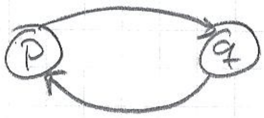
Anonymous code / Anonym kod: DAT060-25

Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare): 7

Consecutive page no. / Löpande sid nr: 2

Question no. / Uppgift nr: 2

$(Fp \wedge Fq) \to F(p \wedge q)$ is not valid. See the model below satisfying the LHS but not the RHS.



✓

However if $Fp \wedge Fq$ is satisfied, it means that regardless of which state we are in, we will eventually reach p and eventually reach q. Therefore, if we are in a state satisfying p, Fq holds and if we are in a state satisfying q, Fp holds.

Combine these and we see that $Fp \wedge Fq$ tells us that we will eventually reach a state satisfying p - and this state will satisfy Fq - or we will eventually reach a state satisfying q - which in turn satisfies Fp.

Hence $(Fp \wedge Fq) \to (F(p \wedge Fq) \vee F(q \wedge Fp))$ holds.

−1, vague, you should reason about paths.

CHALMERS Anonymous code / Anonym kod: DAT060-25 | Points for question / Poäng på uppgiften: 3 | Consecutive page no. / Löpande sid nr: 3 | Question no. / Uppgift nr: 3

One binary predicate symbol R
One unary function symbol f
One constant c

a) A model of this language is: A non empty universe A.
$R^M \subseteq A \times A$
$f^M \subseteq A$
$c \in A$
And a lookuptable l

✓

b) $R(c, c) \to \forall x\, R(x, f(x))$

- 2 missing

3

CHALMERS	Anonymous code / Anonym kod	Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av läraro)	Consecutive page no. / Löpande sid nr 4
	DAT060-25		Question no. / Uppgift nr 4

a) $\exists x(P(x) \land \neg M(x)), \exists y(M(y) \land \neg S(y)) \vdash \exists z(P(z) \land \neg S(z))$

$A = \{0, 1\}$
$P^M = \{0\}$
$M^M = \{1\}$
$S^M = \{0\}$ ✓

$\left.\begin{array}{l} P(0) \land \neg M(0) = T \\ M(1) \land \neg S(1) = T \end{array}\right\}$ Premise true

$\left.\begin{array}{l} P(0) \land \neg S(0) = F \\ P(1) \land \neg S(1) = F \end{array}\right\}$ conclusion false

b) $\forall x \neg R(x,x) \vdash \forall x \forall y (R(x,y) \to \neg R(y,x))$

A = All cousines in the world.
$R^M = \{(x,y) \mid x, y \in A,\ x$ and y are cousines$\}$ ✓

You are never your own cousine, hence $R(x,x) = F$ and $\neg R(x,x) = T$ for all x.

If x is cousine to y then it must be the case that y is cousine to x, hence $R(x,y) \to \neg R(y,x)$ is false for all x and y.

c) $\forall x \forall y (R(x,y) \to \neg R(y,x)) \vdash \neg R(z,z)$

1	$\forall x \forall y (R(x,y) \to \neg R(y,x))$	Premise
2	x_0 $\forall y (R(x_0,y) \to \neg R(y,x_0))$	$\forall x$e 1
3	$R(x_0,x_0) \to \neg R(x_0,x_0)$	$\forall y$e 2
4	$R(x_0,x_0)$	assumption
5	$\neg R(x_0,x_0)$	$\to$e 4,3
6	$\bot$	$\neg$e 5,4
7	$\neg R(x_0,x_0)$	$\neg$i 4-6
8	$\forall z \neg R(z,z)$	$\forall$i 2-7 ✓

d) /

Answer only one question on this page. Do not write on the back of this paper

The set $\{\wedge, \neg, \vee, \rightarrow\}$ is the full toolbox. If we can use $\wedge$ and $\neg$ to show $\vee$ and $\rightarrow$, $\wedge$ and $\neg$ are adequate.

p	q	$p \vee q$	$\neg(\neg p \wedge \neg q)$	$p \rightarrow q$	$\neg(p \wedge \neg q)$
0	0	0	0	1	1
0	1	1	1	1	1
1	0	1	1	0	0
1	1	1	1	1	1

So: $p \vee q \equiv \neg(\neg p \wedge \neg q)$
$p \rightarrow q \equiv \neg(p \wedge \neg q)$

Hence $\{\wedge, \neg\}$ is adequate.

$\forall x \neg R(x, x)$
$\forall x \forall y \forall z. ((R(x, y) \wedge R(y, z) \to R(x, z))$
$\forall x \exists y\, R(x, y)$
$\forall x \exists y\, R(y, x)$
$\forall x \forall y (R(x, y) \to \exists z (R(x, z) \wedge R(z, y)))$

Let $A = \mathbb{Q}$
$R^{M} = \{(x, y) \mid x, y \in \mathbb{Q},\ x < y\}$

(every number only apear once)

There will be no douplets in A, hence $\forall x \neg R(x, x)$ is satisfied.
Integers obey the ordered relation, hence $\forall x \forall y \forall z ((R(x, y) \wedge R(y, z) \to R(x, z))$ is satisfied.
Since $\mathbb{Q}$ is infinite "in both directions" $\forall x \exists y R(x, y)$ and $\forall x \exists y R(y, x)$ are satisfied.
Since we can always squeeze another number in between two numbers x and y we know that $\forall x \forall y (R(x, y) \to \exists z (R(x, z) \wedge R(z, y)))$

a) $\forall x(P(x) \to (Q(x) \vee R(x))), \neg\exists x(P(x) \wedge R(x)) \vdash \forall x(P(x) \to Q(x))$

1.	$\forall x(P(x) \to (Q(x) \vee R(x)))$	premise
2.	$\neg\exists x(P(x) \wedge R(x))$	premise
3	x_0	
4	$P(x_0)$	assumption
5	$P(x_0) \to (Q(x_0) \vee R(x_0))$	$\forall x$e 1
6	$Q(x_0) \vee R(x_0)$	$\to$e 4,5
7	$Q(x_0)$	assumption
8	$R(x_0)$	assumption
9	$P(x_0) \wedge R(x_0)$	$\wedge$i 4,8
10	$\exists x(P(x) \wedge R(x))$	$\exists$i 9
11	$\perp$	$\neg$e 2,10
12	$Q(x_0)$	$\perp$e 11
13	$Q(x_0)$	$\vee$e 6, 7-7, 8-12
14	$P(x_0) \to Q(x_0)$	$\to$i 4-13
15	$\forall x(P(x) \to Q(x))$	$\forall x$i 3-14 ✓

b) $\forall x(f(f(x)) = x) \vdash \forall x \exists y\, (x = f(y))$ 0

1	$\forall x(f(f(x)) = x)$	
2	x_0 $f(f(x_0)) = x_0$	$\forall x$e 1
3	$\neg\exists y\ x_0 = f(y)$	assumption
4	y_0 $x_0 = f(y_0)$	assumption
5	$\exists y\ x_0 = f(y)$	$\exists$i 4
6	$\perp$	$\neg$e 5,3
7	$\perp$	$\exists$e 3, 4-6 not an existential quantifier
8	$\exists y\ x_0 = f(y)$	PBC
	$\forall x \exists y\ x = f(y)$	$\forall$i 2-8